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Aerodynamic lift and moment fluctuations of a sphere

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Measurements are reported of the fluctuating lift acting on a sphere and the moment acting about the centre of a sphere at supercritical Reynolds numbers ($R > 4 \times 10^5$). The lift and moment fluctuations are random functions of time which scale with the free-stream dynamic pressure and sphere dimensions. The power spectra of the lift and moment also scale with the above parameters and with the Strouhal number, nd/U. The spectra contain a maximum spectral density at very low frequencies ($nd/U < 0{\cdot}0003$) and do not reveal appreciable effects of vortex shedding at discrete frequencies.

Hot wire anemometers were placed near the surface but outside the boundary layer along a great circle in the meridian plane in which the lift was measured. The fluctuating velocity component near the surface on the upstream hemisphere in this meridian plane is highly correlated with the fluctuating lift in the same meridian plane. The correlation betweeen the lift and tangential velocity near the surface suggests that the fluctuating lift is produced by the component of fluctuating bound vorticity about the sphere that is normal to the meridian plane in which the lift force is measured. The fluctuating moment measured about an axis passing through the centre of the sphere and perpendicular to the above meridian plane is almost perfectly correlated with the fluctuating lift (the measured correlation coefficients were 0·99 and 1·00). The fluctuating moment coefficient is very small ($\sqrt{\overline{C_m^2}} \simeq 5 \times 10^{-4}$) compared to the fluctuating lift coefficient ($\sqrt{\overline{C_L^2}} \simeq 6 \times 10^{-2}$). The exceptional correlation between the random lift and moment suggests that the unsteady moment about the sphere centre (which can be produced only by shear stress fluctuations) is caused by the fluctuations of bound vorticity (residing in the boundary layer and wake) that are responsible for the unsteady lift.

1. Introduction

The sphere is a perfectly symmetrical bluff body which, nevertheless, experiences unsymmetric aerodynamic forces transverse to the direction of motion when the Reynolds number is large enough to allow the wake to become unstable. The unsteady wake flow causes fluctuating aerodynamic forces and moments that affect the motion of the sphere when it moves freely through a fluid. Newton (1726) reported the experiments of Dr Desaguliers who, in 1719, measured sphere drag by timing the fall of spherically shaped inflated hogs' bladders, 5 in. in diameter, from the 272 ft. high copula of St Paul's Church in London. In some of the tests Newton reported that 'the bladders did not always fall directly down, but sometimes fluttered a little in the air and waved to and fro as they were descending'. Since Newton's time there have been numerous observations of the unsteady motion of spheres in free flight in air and in liquids.

Robins (1805) speculated that the random whirling motion of spherical shot upon emission from a cannon barrel caused deviation of the shot along a line oblique to the expected course and reports deviations of as much as 15° between two trajectories under the same conditions. Robins also thought that the deviation was similar to the transverse motion experienced by a spinning pendulum bob or by a 'cut' or 'sliced' tennis ball. It was known at that time that spiral grooves in a cannon barrel, which impart a considerable spin along an axis parallel to the line of flight, could reduce the random whirling of the shot so that more accurate trajectories were obtained. Rayleigh (1887) attributed the irregular flight of a tennis ball to the lift caused by circulation produced when the ball is 'cut' or 'sliced'. In effect he attributed the lift to the Magnus (1853) effect as applied to a three-dimensional body. Maccoll (1928) performed experimental measurements of the lift produced by a sphere spinning about a cross-wind axis in a uniform stream. The lift developed was considerable at high rotation speeds ($C_{L_{max}} \simeq 0{\cdot}4$).

When the sphere does not spin, random irregular motions are observed. Schmidt (1919), using a multiple exposure camera, observed the initial stages of the motion of spheres falling in liquids and found that the spheres did not fall vertically and that the vertical acceleration of the sphere fluctuated. He observed changes in the falling speed and in the direction of motion in a horizontal plane and mentioned that the fluctuations of the motion appeared to be related to changes in the wake configuration (eddy shedding) as revealed by dyed fluid in the wake. Many other people in the time period 1920 to 1959 have reported observations of unsteady wakes and motion of spheres. In general these observations are of a qualitative nature and no detailed investigations of the transverse sphere motions before 1959 have been discovered. The review paper of Torobin & Gauvin (1959) contains an extensive list of references that will serve as an introduction to this literature.

During the past decade there has been an increased interest in the irregular random motion of spheres falling or rising freely in air or liquid. The interest in these motions is connected with balloon astronomy, meteorological balloons and wind profile measurements using balloons. Investigations of the random motions of rising spheres have been reported by Preukschat, MacCready & Jex and Scoggins.

Preukschat (1962) described the unsteady motions of rising spheres and demonstrated that the amplitude of transverse motions depends on the ratio of sphere mass to mass of fluid displaced by the sphere. The transverse motions are of small amplitude if the ratio is of order one or greater but of increasing amplitude as the ratio is reduced.

In an excellent paper, MacCready & Jex (1964) studied the unsteady motion of falling and rising spheres in the Reynolds number range $10^4 < R < 10^6$. They give lucid qualitative descriptions of the flow field, wake and interactions between the sphere motion and wake that cause the random motion of rising or falling spheres. According to MacCready & Jex, the amplitude of the transverse motions of a sphere is much greater if the Reynolds number is supercritical. MacCready & Jex give simple qualitative explanations of the complex coupling or interaction between transverse sphere motion, rotational sphere motions and the unsteady wake flow behind the sphere.

Scoggins (1964, 1967) has also reported that the unsteady transverse motions of rising balloons of near spherical shape are much greater when the Reynolds number is supercritical, $R_{crit} \simeq 4 \times 10^5$. Scoggins (1964) reported tests of a spherical balloon with large roughness elements (conical paper cups) glued to the surface and showed that the random irregularities of the motion of the rising balloon are much reduced in the supercritical range of Reynolds number. He also, Scoggins (1967), reports radar trajectory data for spherical balloons rising in the atmosphere. He made simplifications to the equations of motion for the rising sphere treating it as a buoyant particle of mass equal to the sphere mass plus apparent mass and subject to unknown forces. Using a computer program containing the simplified equations of motion and the radar trajectory data Scoggins computed the statistical properties of the unsteady lift and drag forces acting on a smooth sphere. His results show that the lift and drag are indeed random functions in which there is no preferrred orientation for the lift force. Scoggins's (1967) results span the subcritical and supercritical régime of Reynolds number and in some cases exhibit a great deal of scatter. The cause of the scatter is not clear, but is no doubt related to the well-known uncertainty produced by differentiation of trajectory data, the unavoidable change in Reynolds number, and the errors inherent in the statistical analysis of finite length records of the random sphere motions.

In an effort to reveal the aerodynamic phenomena responsible for the unsteady motions of a sphere we wish to report measurements of the fluctuating lift and moment acting on a fixed sphere in a low turbulence wind tunnel. In this investigation the additional complications of sphere motion in response to unsteady forces and free-stream turbulence have been suppressed. The unsteady forces and moments acting on the sphere are caused only by fluctuations in the wake flow field. Measurements of the correlation coefficient between the fluctuating lift, moment and velocity near the sphere surface have been used to give a qualitative understanding of the gross features of the fluctuating flow field. The sphere lift and moment measurements are all made in the supercritical régime ($R > R_{crit}$). The Reynolds number range is $4{\cdot}5 \times 10^5 < R < 1{\cdot}7 \times 10^6$. The surface of the sphere is rough thus ensuring that the boundary layer is turbulent ahead of the separation line.

2. Experimental apparatus

The experiments were conducted in the subsonic wind tunnel at the Aerospace Engineering Laboratories of The University of Michigan. This closed circuit tunnel has a 5 by 7 ft. test section with continuously variable speed up to 225 ft./

sec. The turbulence level in the test section is 0·1 % axially and 0·13 % transversely at 50 ft./sec. At 150 ft./sec the maximum speed used for these tests, the axial and transverse components increase to 0·14 and 0·22 % respectively.

The test model, which was designed to have as small a mass and moment of inertia as possible, was a 1 in. thick spherical shell machined from Dow Corning styrofoam. It was constructed in two hemispherical halves and mounted on a magnesium frame $\frac{1}{32}$ in. thick. The halves were joined by a metal-to-metal contact within the sphere, and the joint was covered with tape, visible in figure 1, plate 1. (Two other strips of tape, one of which may be seen in figure 1, cover glued joints made necessary by the maximum size of styrofoam billets available.) The final streamwise outer diameter was 24·00 in. with a cross-stream diameter of 23·81 in. The surface of the machined styrofoam consists of a random pattern of closely spaced airtight holes with an average diameter of 0·06 in. and depth of 0·06 in.

The ratio of the frontal area of the sphere to empty test section area is $\frac{1}{35}\pi$. It is necessary to consider the effect that the constraint of the wind-tunnel walls will have on the flow over the sphere. The theory of the blockage effect on bluff bodies in a closed wind tunnel has been studied by Maskell (1963). Maskell recommends that for practical purposes the effect of blockage on the drag coefficient in a steady flow may be estimated from the relation

$$(C_D - C_{D_C})/C_D \simeq (\epsilon C_D S)/C,$$

where $\epsilon = \frac{5}{2}$ for a symmetrical three-dimensional bluff body, C_D is the drag coefficient in the wind tunnel, C_{D_C} is the correct drag coefficient in an infinite medium, S is the frontal area of the body and C is the cross-sectional area of the empty test section. Taking $C_D \simeq 0{\cdot}5$ for a sphere at $R = 10^6$, the effect of the wind tunnel wall constraint is to cause an increase in measured sphere drag coefficient over the value that would be obtained in an infinite stream such that

$$(C_D - C_{D_C})/C_D = 0{\cdot}11.$$

The increase in drag owing to the constraint of the wind-tunnel walls is caused by an increase of the velocity near the body to higher values than would be obtained in an infinite stream. The dominant effect of the walls is usually taken to be equivalent to a simple increase in free-stream velocity of an unconfined flow and it is assumed that the pressure coefficient at any point on the body is unchanged. The increase in free-stream velocity is related in part to the volume distribution of the body (solid blockage) and in part to the displacement effect of the wake (wake blockage). In considering wake blockage effects Maskell has also included an (empirical) correction for the shrinkage of the wake caused by the constraint of the walls.

With regard to blockage corrections for an unsteady flow over a bluff body virtually nothing is known. Very severe constraint by the walls would seriously affect the unsteady wake motion and vortex shedding processes. However, the steady drag coefficient is only increased by approximately 10 % indicating (we believe) that the constraining effect of the walls is not great enough to seriously affect the fundamental processes of vortex shedding that produce the lift and moment fluctuations that we have observed.

The sphere was sting mounted in the tunnel. The sting was made from a 10 ft. length of 2 in. diameter cold rolled steel tubing with $\frac{1}{4}$ in. wall thickness, and was supported on the centreline of the tunnel test section by twelve $\frac{1}{8}$ in. diameter aircraft cables (see figure 1, plate 1). Both lift and moment measurements were made at several Reynolds numbers. The moment about the centre of the sphere was measured with a dynamic-moment balance mounted on a yoke fastened to the sting inside the sphere. A sketch of the balance is shown in figure 2.

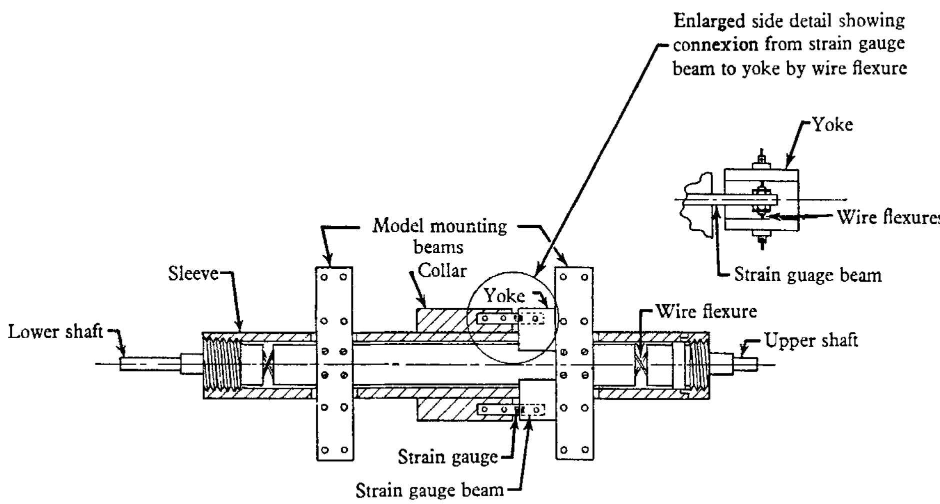


FIGURE 2. Sketch showing essential components of moment balance, $\frac{1}{2}$ scale.

The balance consists of an outer cylindrical shell which contains an inner shaft supported by a pair of three-wire flexures. These flexures offer minimum resistance to relative rotation between shaft and shell without the disadvantages of ordinary bearings which are subject to effects of dust, corrosion and wear. The torque which acts on the disk is carried by the model mounting beams to the inner shaft. This torque is then transmitted to the strain gauge beams by the yokes and 0·015 in. steel flexure wires, and finally carried to the upper and lower shafts by the components of the outer shell. The gauge beam design required a compromise between the necessity for a high moment balance sensitivity on one hand and stiffness on the other. High sensitivity is needed to measure small aerodynamic moments accurately, and stiffness is required to prevent low-frequency vibration of the disk relative to the supporting shaft. The gauge beams are made from steel hardened to Rockwell C 40. The 350 ohm, epoxy backed, foil guages* are of constantan alloy compensated for steel and bonded to the metal by an epoxy heat-curing cement.† The four gauges (two gauges per beam) are connected in a four-arm bridge circuit driven by a carrier amplifier system‡ whose output was amplified with another amplifier.§ The frequency response of

* Budd Instruments Division, type C6-161-B350.
† Wm. T. Bean Co., type BAP-1.
‡ Consolidated Electrodynamics Corp., Amp. System D.
§ Dana DC amplifier Model 3520, V-2.

the system was constant from 0 to 600 Hz. It was experimentally determined that the minimum readable moment with this system is of order 0·01 ft. lb. The balance was tested for interaction with the expected maximum drag load (26 lb.) applied normal to the axis of rotation. The interaction was of the order of the minimum measurable moment. A typical calibration is given in figure 3.

To ensure that the moment balance was mounted on an axis passing through the centre of the sphere, a static test was performed by submerging a portion of the upstream and cross-stream sphere surface in a tank of water. The sphere was

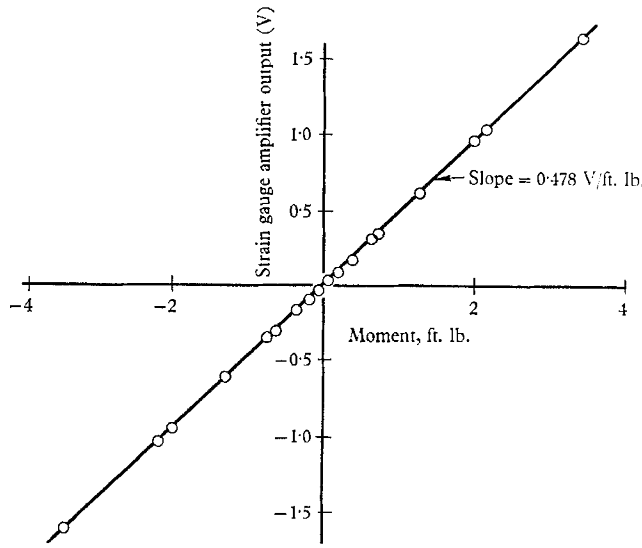


FIGURE 3. Typical static calibration of moment balance.

first covered with a thin film of polyethylene plastic to keep it dry. The moment produced by hydrostatic pressure forces on a sphere should be zero. The output of the moment balance was very small but not zero. From the balance output and the volume of fluid displaced we found by calculation that the moment axis was displaced downstream from the sphere centre 0·087 in. and in the cross-wind direction less than 10^{-3} in. These small offsets will, however, allow lift and drag forces to contribute to the moment measured by the balance. Because the purpose of the moment balance was to measure moments due to shear stresses only, it is desirable to estimate the error in the measured moment caused by the effects of the lift and drag. Denoting the measured moment fluctuations by M, the lift and drag fluctuations by L and D, and the moment fluctuation caused by shear stresses by M_s, we write

$$M = L\delta_1 + D\delta_2 + M_s, \tag{1}$$

where the sign convention for lift and moment is given in figure 8. In dimensionless form (1) becomes

$$C_M = C_L\frac{\delta_1}{d} + C_D\frac{\delta_2}{d} + C_{M_s}, \tag{2}$$

where d is the sphere diameter and δ_1 and δ_2 are the appropriate offsets. Hence

$$\overline{C_M^2} = \overline{C_L^2}\left(\frac{\delta_1}{d}\right)^2 + \overline{C_D^2}\left(\frac{\delta_2}{d}\right)^2 + \overline{C_{M_s}^2} + 2\frac{\delta_1\delta_2}{d^2}R_{LD}\sqrt{\overline{C_L^2}}\sqrt{\overline{C_D^2}} + \frac{2\delta_1}{d}R_{LM_s}\sqrt{\overline{C_L^2}}\sqrt{\overline{C_{M_s}^2}} + \frac{2\delta_2}{d}R_{DM_s}\sqrt{\overline{C_D^2}}\sqrt{\overline{C_{M_s}^2}}, \quad (3)$$

where R_{LD} denotes the correlation factor between the fluctuating lift and drag signals, etc. The measured values of $\sqrt{\overline{C_M^2}}$ and $\sqrt{\overline{C_L^2}}$ were 5×10^{-4} and 6×10^{-2} (see figures 4 and 6). Assuming that $\overline{C_D^2}$ is of the order of $\overline{C_L^2}$, and assuming that all correlations were one, equation (3) was solved for the ratio of the moment due to shear stresses to the total measured moment. There are two solutions. Only one solution,

$$\sqrt{\overline{C_{M_s}^2}}/\sqrt{\overline{C_M^2}} = 0{\cdot}65, \quad (4)$$

has physical significance. This means that the measured root-mean-square moment coefficient is at most 50 % greater than the root-mean-square moment coefficient caused by shear stress.

With the moment balance installed, the natural frequency of sphere, mounting brackets and central cylinder of the balance in free oscillation about the axis of the sphere was found to be 7·7 Hz. Since this frequency falls within the anticipated frequency range of the unsteady moments it was necessary to compensate the moment signal for the spurious signals caused by balance oscillations using an analogue-computer* circuit similar to that described in Willmarth *et al.* (1967).

To measure the steady lift force on the sphere, the stress in the sting was measured with four strain gauges† mounted in pairs on either side of the sting at two stations 0·5 ft. apart. The strain gauges were connected in a bridge circuit powered by a regulated d.c. voltage‡ and the differential output of the bridge was amplified by a d.c. amplifier§ whose frequency response was uniform from 0 to 50 KHz. In this configuration the natural frequency of the sting and sphere in lateral bending oscillations was approximately 36 Hz. A static calibration of the lift and moment balance was made by applying horizontal forces (produced by balance weights) to a thread attached to the stagnation point of the sphere before and after each run. In all cases the calibration was linear and repeatable. A special plug-in unit‖ for the oscilloscope¶ was used to obtain a crude spectrum analysis of the lift force signal. The natural frequency of the beam and sphere appeared to be sufficiently above the frequency range of the lift spectrum to allow adequate filtering of the resonance peak using a simple low pass filter and it was decided that a compensation circuit would not be necessary.

Additional strain gauges were also mounted on the sting in an attempt to obtain a second measurement of the moment about the sphere centre. However, the small magnitude of the moment signal relative to electronic noise and vibration signals from the tunnel structure made this method less reliable than the

* Applied Dynamics Model AD-1 electronic differential analyser.
† Kulite Semiconductor Strain Gauges, Type DDN-350-500.
‡ Trygon Model HR 40-750.
§ Dana DC amplifier model 3520 V-2.
‖ Tektronix Type 1L5.
¶ Tektronix Type 554.

direct moment measurements described above. The results did, however, verify the order of magnitude of the moments obtained directly with the dynamic moment balance.

The compensated moment signal and the uncompensated lift signal were recorded at several Reynolds numbers on an Ampex Model FR-1100 frequency modulated tape recorder. The data was recorded at $7\frac{1}{2}$ in./sec tape speed in the frequency band 0 to 1250 Hz and played back at 60 in./sec tape speed. A low frequency variable-band-width wave analyzer consisting of a narrow-band filter circuit made up from analogue computer elements with centre frequency range from 0·16 to 160 Hz was used to obtain power spectra of the moment and lift force. By playing the data back at 60 in./sec, power-spectral-density measurements (which were corrected to yield spectral density per unit band-width) were made at frequencies as low as 0·02 Hz.

Root-mean-square amplitudes of the lift and moment signals were determined using the analogue computer. A correlation function computer* with a 20 sec time constant was used to determine the correlation coefficients between lift, moment and velocity. Detailed descriptions of the electronic circuitry and the special analogue-computer circuits are given by Willmarth & Hawk (1964).†

3. Discussion of measurements

3.1. *Lift fluctuations*

The lift fluctuations were measured in a horizontal plane. From symmetry it can be shown that random lift fluctuations on the sphere will have no preferred orientation in a plane normal to the free-stream direction (Scoggins (1967) also presents experimental verification) so that it is only necessary to measure the fluctuating component of the lift in a single meridian plane.

The lift fluctuation signal displayed on the oscilloscope appeared random in time and contained large amplitude variations at very low frequencies. We often observed that the lift would rapidly become negative (or positive) with large amplitude and would remain negative (or positive) for many seconds. Superimposed on the large amplitude excursions were smaller amplitude fluctuations at considerably higher frequencies.

The root-mean-square lift coefficient based on frontal area of the sphere was measured at three different Reynolds numbers. The results, shown in figure 4, reveal that the root-mean-square lift coefficient

$$\sqrt{\overline{C_L^2}} = \sqrt{\overline{L^2}}/q\pi(\tfrac{1}{4}d^2) \tag{5}$$

(where q is the free-stream dynamic pressure) was constant for

$$5\times 10^5 < R < 1{\cdot}8\times 10^6.$$

The power spectrum of the fluctuating lift is

$$f_L(\omega) = \frac{1}{2\pi}\int_0^\infty \overline{L(t)\,L(t+\tau)}\cos\omega\tau\,d\tau \tag{6}$$

* Princeton Applied Research Corp. Model 100 Signal Correlator.

† If this reference is not available to the reader, one of us (W. W. W.) will be happy to supply any additional information required.

and the autocorrelation of the lift is

$$\overline{L(t)\,L(t+\tau)} = \int_0^\infty f_L(\omega)\cos\omega\tau\,d\omega. \tag{7}$$

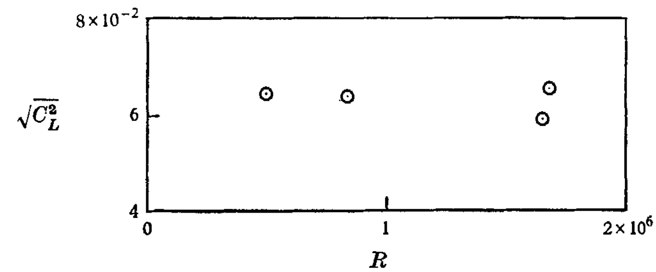


FIGURE 4. Root-mean-square lift coefficient of the sphere as a function of Reynolds number.

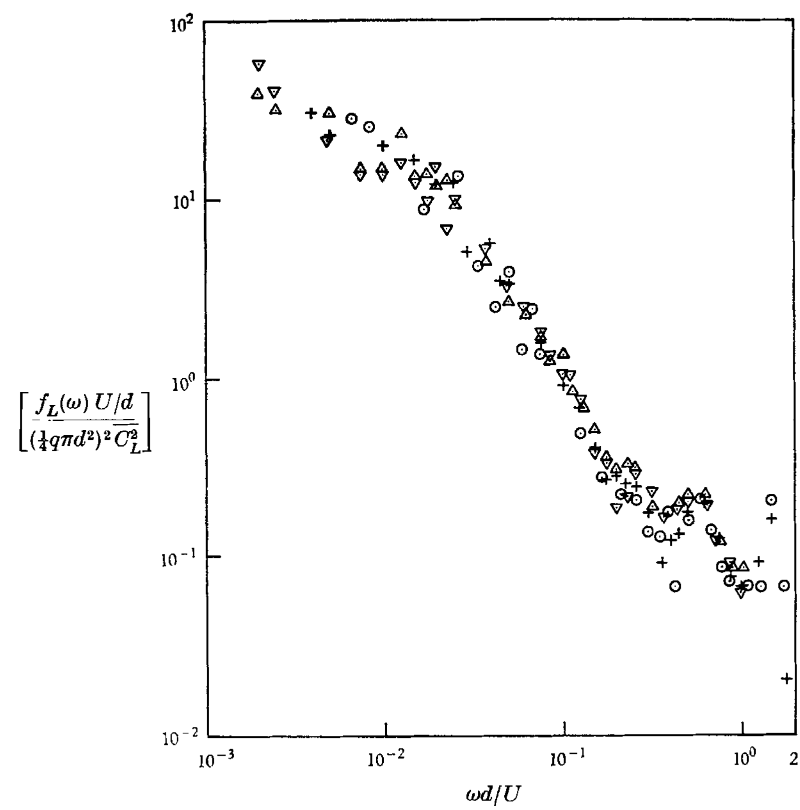


FIGURE 5. Dimensionless power spectra of the fluctuating lift acting on the sphere. ⊙, $R = 4{\cdot}84\times10^5$; ▽, $R = 8{\cdot}26\times10^5$; △, $R = 16{\cdot}67\times10^5$; +, $R = 16{\cdot}46\times10^5$.

Figure 5 shows the results of measurements of the normalized power spectrum of the lift plotted in dimensionless form, the spectra scale with the Strouhal number. There are two interesting features of these spectra. First, the spectral density is a maximum at the lowest frequency, $\omega d/U \simeq 2\times10^{-3}$, and second, no significant energy is present at discrete frequencies at these supercritical Reynolds numbers. The slight bump in the spectra at $\omega d/U \simeq 0{\cdot}6$ corresponds to a frequency of $0{\cdot}095U/d$. Kendall (1964) reports definite periodicity in the turbulent wake at a frequency of $0{\cdot}18U/d$ at Reynolds numbers between 400 and 40000. We do not know the cause of the slight bump observed at a frequency of $0{\cdot}095U/d$. The energy content of the fluctuation at this frequency is not significant and is too small to allow us to investigate the phenomenon.

3.2. *Moment fluctuations*

The moment fluctuation measurements include the root-mean-square moment coefficient about a vertical axis passing through the centre of the sphere,

$$\sqrt{\overline{C_m^2}} = \sqrt{\overline{M^2}}/q\pi(\tfrac{1}{4}d^3) \tag{8}$$

and the power spectrum of the moment

$$f_M(\omega) = \frac{1}{2\pi}\int_0^\infty \overline{M(t)\,M(t+\tau)}\cos\omega\tau\,d\tau, \tag{9}$$

where the inverse transform of the moment spectrum is defined in the same way as for the lift spectrum, see (7).

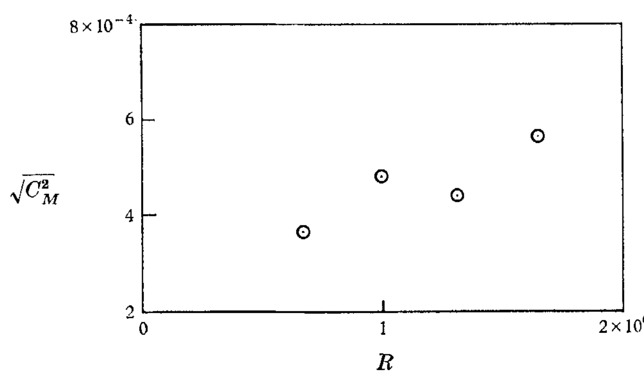


FIGURE 6. Root-mean-square moment coefficient about the centre of the sphere as a function of Reynolds number.

The results of the moment measurements are shown in figures 6 and 7. The root-mean-square moment is extremely small, $\sqrt{\overline{C_M^2}} \simeq 5\times 10^{-4}$. This value of the root-mean-square moment represents an upper bound. A lower bound is given by (4) so that the root-mean-square moment coefficient caused by shear stress alone is in the range $3{\cdot}25\times 10^{-4} \leqslant \sqrt{\overline{C_{M_s}^2}} \leqslant 5\times 10^{-4}$.

The results of measurements of normalized power spectra of the moment are plotted in figure 7 in dimensionless form, the moment spectra scale with the Strouhal number. The moment spectra have almost the same shape as the lift spectra, see figure 5. This is not surprising in view of the results of our measurements of correlation between lift, moment and the velocity tangent to the sphere surface discussed below. When we examined the error produced by the misalignment of the moment balance axis relative to the sphere centre, see (3), we found that the largest contribution to the error in the moment signal comes from the term $L\delta_1$ in (1), where $\delta_1 = 0{\cdot}087$ in. is the offset in the stream direction. Since the lift and moment spectra have the same shape, the true moment spectra have the shape of the measured moment spectra and may be less than the measured spectra shown in figure 7, but, according to (4), the true spectra must be greater than 0·65 times the measured spectra.

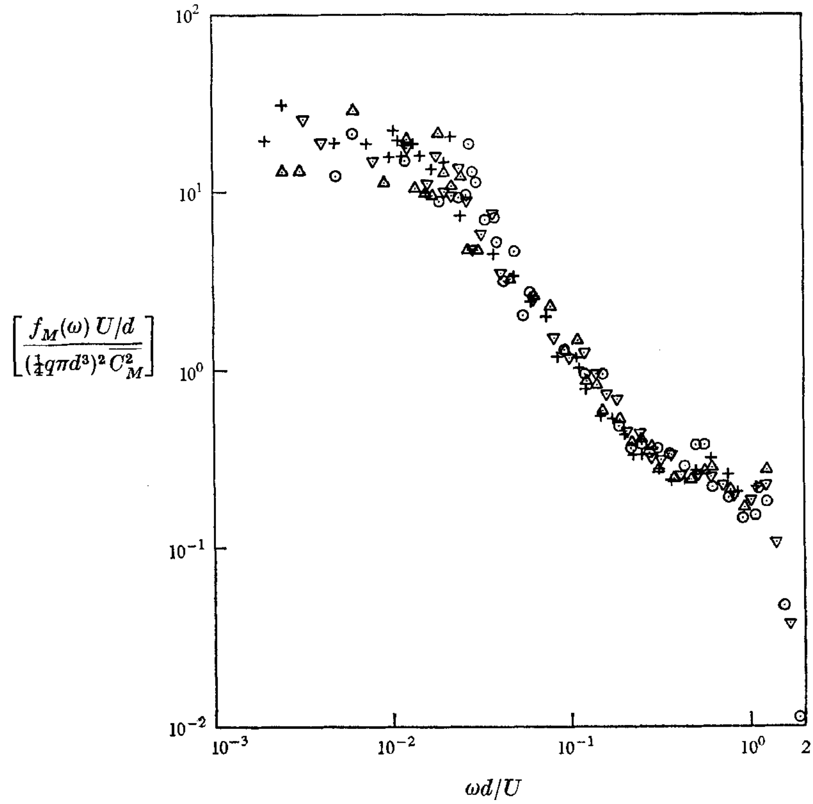


FIGURE 7. Dimensionless power spectra of the fluctuating moment about the centre of the sphere as a function of Reynolds number. ⊙, $R = 6{\cdot}65 \times 10^5$; ▽, $R = 9{\cdot}95 \times 10^5$; △, $R = 13{\cdot}13 \times 10^5$; +, $R = 16{\cdot}49 \times 10^5$.

3.3. *Correlations between lift and moment*

A signal correlator,* operating in the a.c. coupling mode, was used to measure the correlation coefficient

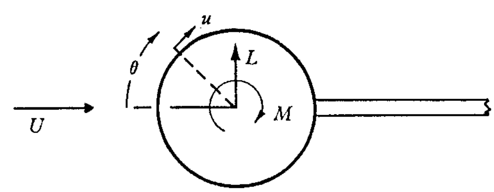
$$R_{LM} = \overline{L(t)\,M(t)}/\sqrt{\overline{L^2}}\sqrt{\overline{M^2}} \tag{10}$$

between lift and moment. For these measurements the small offset in the stream direction, $\delta_1 = 0{\cdot}087$, see (1), was removed by using shims at the attachment brackets of the moment balance. The correlation was measured in a band width of approximately $0{\cdot}026 < \omega d/U < 1{\cdot}57$ so that all but 40 % of the spectral energy contained in the lowest frequencies is included. The results of two correlation

* Princeton Applied Research Model 100.

measurements were $R_{LM} = 1{\cdot}0$ and $R_{LM} = 0{\cdot}99$. The configuration of the sphere and the lift and moment sign convention is given in figure 8.

We found by observing simultaneous oscilloscope traces of lift and moment signals that the lift and moment were correlated at all frequencies lower than $\omega d/U = 0{\cdot}026$ but it was very difficult and time consuming to attempt quantitative measurements of the correlations at dimensionless frequencies less than $\omega d/U = 0{\cdot}026$ because the time constant (20 sec) built into the averaging circuits of the signal correlator is rather short relative to the much longer time constant of the large amplitude low frequency lift and moment signals. To effectively increase the time constant of the correlator we recorded the output signal from



θ ...	30°	−30°	−60°	90°	140°	−140°
R_{Lu}	0·69	0·74	0·77	0·80	0·11	0·14
R_{Mu}	0·78	0·67	—	—	—	—
R_{uu}	0·80 (30° and −30°)		—	—	—	—
R_{LM}	0·99 and 1·00 (all θ)					

All Correlations measured in the frequency band
$0{\cdot}026 < \omega d/U < 1{\cdot}57$

FIGURE 8. Sketch of sign convention for lift, moment and velocity correlations. Summary of measured correlation coefficients.

the correlator averaging circuit on a strip chart recorder for periods of the order 10 min. This was approximately the maximum averaging time that could be used because it required 30 min to measure one correlation coefficient since $\overline{LM}$, $\overline{M^2}$ and $\overline{L^2}$ had to be measured and during a time of the order of 30 min appreciable drift of the carrier amplifier was observed. Thus, there was unavoidable drift in the system resulting in inaccurate correlation coefficients. For this reason the correlation measurements were made with a.c. coupling in which the drift signals and the fluctuations at dimensionless frequencies less than $\omega d/U = 0{\cdot}026$ were removed.

3.4. *Correlations between lift and velocity and moment and velocity*

Hot wires operated in the constant temperature mode using transistor amplifiers* were mounted at a few points on the sphere just outside the boundary layer $\frac{7}{16}$ in. from the surface. The wires were located in the horizontal meridian plane (in the same plane that the lift was measured). Correlation coefficients

$$R_{Lu} = \overline{L(t)\,u(t)}/\sqrt{\overline{L^2}}\sqrt{\overline{u^2}} \tag{11}$$

* L. T. Miller, Model M-5.

and $$R_{Mu} = \overline{M(t)\,u(t)}/\sqrt{\overline{M^2}}\sqrt{\overline{u^2}} \quad (12)$$

between the hot-wire velocity signal and the lift and moment balance signals were measured with the signal correlator. The correlation coefficients were measured in the restricted frequency range, $0{\cdot}026 < \omega d/U < 1{\cdot}57$, for the same reasons discussed above in § 3.3.

The values of the correlation coefficients are shown in a table, see figure 8.* The correlation between the potential flow velocity field over the upstream hemisphere and the random lift and moment is very large. The correlation between the velocity at $\theta = \pm 140°$ in the wake and the lift and moment is not as large.

Our interpretation of the correlation measurements is that they indicate that large-scale spatially coherent velocity fluctuations in the potential flow about the sphere are responsible for the sphere lift and moment. These large-scale velocity fluctuations can be interpreted as fluctuations in bound vorticity that are driven by the random turbulent flow in the wake. If the fluctuating bound vorticity is regarded as a random vector passing through the centre of the sphere, the component of this bound vorticity vector that is normal to a given meridian plane is responsible for the lift fluctuations in that meridian plane. This interpretation is suggested by Rayleigh's (1887) statement that a 'cut' or 'sliced' tennis ball develops lift produced by circulation caused by a steady spin and by the experiments of Maccoll (1928). In the present case the sphere does not spin; however, there is a considerable fluctuating lift that is highly correlated with the unsteady circulation about the fixed sphere.

3.5. *Correlation between velocities on either side of the stagnation point and in the wake*

The correlation coefficient, R_{uu}, between the tangential velocities in a meridian plane at two points on either side of the stagnation point, $\theta = \pm 30°$, was also measured. The correlation coefficient was large, $R_{uu} = 0{\cdot}8$; figure 8 gives the direction of positive velocity, moment and lift.† This measurement was made to emphasize the relation between the present investigation of unsteady lift and our earlier investigation of stagnation point wandering (Kuethe, Willmarth & Crocker (1960) in which we found that the stagnation point on a hemisphere-cylinder model moved about in a random manner owing to turbulent velocity fluctuations in the free stream). The difference between the present investigation and that of Kuethe *et al.* (1960) is that in the present investigation the wake behind the sphere, which is quite free to move about, causes the unsteady flow whereas on a hemisphere-cylinder the long cylindrical afterbody constrains the wake. Both investigations show that the unsteady potential flow over the upstream hemisphere varies randomly with time but is spatially well correlated or organized. The power spectra of the fluctuations have very different shapes in the two investigations because the source of the unsteady flow is different. Also the power spectra of velocity fluctuations near the hemisphere-cylinder stagna-

* The slight asymmetry in the correlations R_{Lu} and R_{Mu} at $\theta = \pm 30°$ is probably caused by inaccuracies in the averaging process as discussed in § 3.3.

† Note that for $\theta > 0$ or $\theta < 0$, u is positive in the clockwise direction.

tion point did not scale with the diameter of the hemisphere, when the diameter was varied by a factor of ten, because the free-stream turbulence scale was the same in the wind tunnel regardless of the model size.

It is worth mentioning here that in interpreting the correlation measurements of R_{Lu} and R_{Mu} the mean velocity at $\pm 140^\circ$ from the stagnation point was assumed to be in the upstream direction. That is to say, we assume that since the boundary layer separates at $\theta \simeq 110^\circ$ at supercritical Reynolds numbers the sign of electrical signals representing velocity fluctuations measured by heat loss from the hot wire are opposite to those measured on the upstream hemisphere. With this interpretation there is agreement that the bound vorticity is to be found about the entire sphere. We note that in the wake at $\pm 140^\circ$ from the stagnation point the correlation coefficients R_{Lu} and R_{Mu}, see figure 8, are of much smaller magnitude than on the upstream hemisphere because only a small fraction of the chaotic velocity fluctuations in the wake are correlated with the necessarily large scale, well organized, fluctuating bound vorticity that produces the lift and moment.

4. Conclusions and interpretation of results

We have found that appreciable lift fluctuations ($\sqrt{\overline{C_L^2}} \simeq 6 \times 10^{-2}$) are produced at supercritical Reynolds numbers on a fixed sphere. Accompanying these lift fluctuations are small moment fluctuations ($\sqrt{\overline{C_M^2}} \leqslant 5 \times 10^{-4}$) about the centre of the sphere. Both the lift and moment fluctuations are highly correlated with fluctuating bound vorticity about the sphere in a meridian plane containing the lift vector and normal to the moment vector. We believe that the fluctuating lift, moment and bound vorticity are produced by random changes in the flow configuration of the wake behind the sphere. Prandtl (1952, p. 142) has discussed the production of bound vorticity and lift when a starting vortex is shed from a spinning cylinder as it beings to translate through a fluid. We propose that on the sphere the fluctuating bound vorticity will also be correlated with the shedding of relatively large turbulent eddies behind the sphere. We hope to be able to test this idea by tests in a newly designed towing tank containing a viscous fluid. We plan simultaneous observations of the unsteady lift and flow visualization studies of the wake at relatively low Reynolds numbers when turbulence in the wake near the sphere is not yet severe.

The measurements that we have made in this investigation should be applicable to the problem of transverse motion of spheres in free flight provided that the average density of the sphere is large compared to the density of the fluid through which it moves. If the average sphere density is of the order of the fluid density (as is the case with rising balloons) we expect that complex interactions between the motion of the sphere, its wake, and the lift forces* produced by the wake will introduce coupling between sphere and wake motions. Indeed, the spiralling of spheres observed by MacCready & Jex (1964) may be one manifestation of this coupling phenomena.

* In the case of a rising balloon the term 'lift' refers to a side force and should not be confused with the buoyant lift of a balloon.

It is significant that the lift spectra, see figure 3, contain large contributions to the special density at extremely low frequencies. This corresponds to the observations of balloon motions by Killen (1960) who reported very large-scale transverse motions of rising balloons. During the flight of a balloon Killen (1960) noted that a transverse motion in one direction rapidly terminated and was followed by a transverse motion in another direction with no apparent relation to the previous motion. This 'zigzag' motion is probably related to our observation of very low frequency, large amplitude lift excursions, see our discussion of the lift fluctuation measurements.

The fact that the spectral density of the lift is a maximum at very low frequencies also indicates that changes in the sphere wake configuration have a long time constant. This suggests that the investigation of unsteady motion of falling spheres or the drag forces on accelerated spheres may be difficult in the laboratory where the duration of the falling or accelerated motion is necessarily limited.

The transverse motion of heavy spheres in free flight will be dominated by the low-frequency contributions to the lift spectrum. As an example, in the game of baseball the pitch known as a 'knuckle ball', which is produced by throwing the ball at medium speed with very little spin,* is noted for its unpredictable deviation from the trajectory anticipated by the batter. We suggest that the unpredictable deviation of the 'knuckle ball' is caused by wake driven (hence randomly orientated) fluctuations in bound vorticity that produce lift forces similar to those measured in this investigation. This suggestion does not completely explain the 'knuckle ball' phenomenon since there remains unanswered the question of the role played by the rough seams on the slowly rotating baseball. The seams on the ball might cause premature boundary-layer separation which can produce asymmetry in the flow and wake that may be large enough to produce appreciable lift forces in the same fashion as the random wake driven lift forces that we have described.

On the other hand, one of the reviewers has suggested that it is his impression that the Reynolds number of a 'knuckle ball' is such that the boundary-layer separation will ordinarily be laminar, in contrast to the experimental work that is reported here. This suggests to him that 'the seam is more likely to delay separation by inducing transition when it appears on the upstream half of the sphere, rather than to hasten separation'. A reference to the 'knuckle ball' (Terrell 1959) was also suggested by the reviewer and it appears from this reference that S. Corrsin had already contributed in Terrell's article the suggestion that the irregular motion of the separation line is responsible for the peculiar trajectory and that the seams would be expected to affect the position of the separation line. It appears to us that the question of the Reynolds number of the 'knuckle ball' and the role played by the seams would be an interesting subject for further research.

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* M. E. Benedict, Head Baseball Coach at The University of Michigan (private communication).

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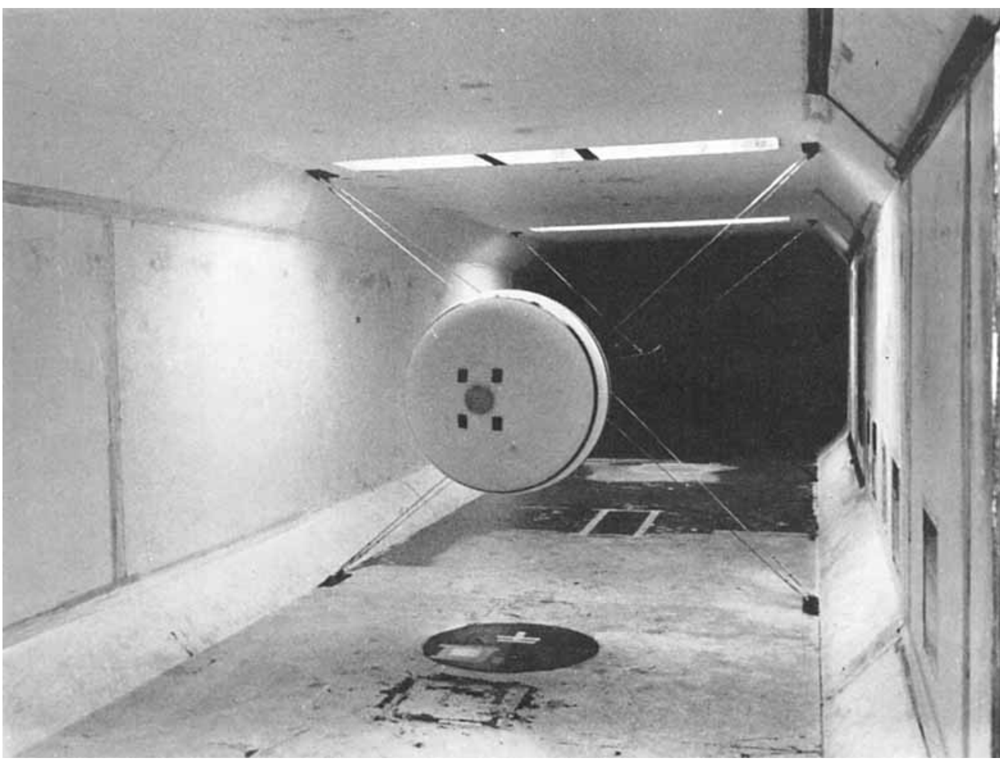

FIGURE 1. Two-foot diameter styrofoam sphere mounted in the tunnel.